Worksheet 3 – Binomial Random Variables

Question 1: Aaron is flipping a coin 30 times. Let X be the number of tails. Find:

A. P(X = 15)
B. P(X = 20)
C. P(X = 4)

Question 2: A baseball player gets up to bat 5 times during a game. His batting average is 0.325. Complete the probability distribution table below and calculate the expected value of X.

X	0	1	2	3	4	5
P(X)						

Question 3: According to New Jersey Transit, the 8:00 am weekday train from Princeton to New York City has a 90% chance of arriving on time on a randomly selected day. Suppose this claim is true. Choose 6 days at random.

A. Find the probability that the train arrives late on exactly two days.
B. Would you be surprised if the train arrived late on 2 or more days? Compute $P(W \geq 2)$ and use this result to support your answer.

Question 4: Suppose that in a certain metropolitan area, 9 out of 10 households have a DVD player. Let X be the number among 4 randomly selected households that have a DVD player.

A. Calculate P(X = 2) and interpret this probability.
B. Calculate P(X=4).
C. Calculate $P(X \leq 3)$.

Question 5: The Los Angeles Times (1992) reported that what airline passengers like to do most on long flight is rest or sleep: in a survey of 3697 passengers, almost 80% did so. Suppose that for a particular route the actual percentage is exactly 80% and consider randomly selecting 6 passengers. Let X be the number among the selected six who rested or slept.

A. Calculate P(X = 4) and interpret the value.
B. Calculate $P(X \geq 4)$.

Question 6: The article "FBI Says Fewer than 25 Failed Polygraph Test" describe the impact of a new program that requires top FBI officials to pass a polygraph test. The article states that false positives (tests in which an individual fails even though he or she is telling the truth) are relatively common and occur about 15% of the time. Suppose that such a test is given to 10 trustworthy individuals.

A. What is the probability that all 10 pass?
B. What is the probability that more than 2 fails, even though all are trustworthy?
C. The article indicates that 500 FBI agents were tested. Consider the random variable X which represents the number of the 500 who fail. If all 500 agents tested are trustworthy, what are the mean and standard deviation of X?
D. The headline indicated that fewer than 25 of the 500 agents tested failed the test. Is this a surprising result if all 500 are trustworthy?

Question 7: You are to take a multiple-choice test consisting of 100 questions with 5 possible responses to each question. Suppose that you have not studied and so you must guess (select one of five answers in a completely random fashion) on each question. Let X represent the number of correct responses on the test.

A. What is your expected score on the exam?
B. Compute the variance and standard deviation of X.
C. Based on your answers to (A) and (B), is it likely that you will score over 50 on this exam? Explain your reasoning.

Question 8: Danny's free throw percentage is 75%. In a particular game, he had 12 free throw attempts and he made only 7. The only question is "How unusual was it for Danny to make at most 7 shots out of 12 attempts?" Find the probability that he made at most 7 out of 12.

Question 9: There are 50 poker chips in a container, 25 of which are red, 15 white, and 10 blue. You draw a chip without looking and then replace it a total of 25 times.

What is the expected number of blue chips you will draw in 25 draws?
What is the standard deviation of the number of blue chips that you will draw?
Is it likely or unlikely that you will draw 9 or fewer blue chips? What about 15 or fewer chips?

Question 10: Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there is a 1-in-38 chance that the ball will land in the "15" slot.

A. How many spins do you expect it to take until Marti wins? Justify your answer.
B. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

Question 11: Anna makes random guesses on a multiple-choice test that has 5 choices for each question. We want to know how many questions she answers until she gets one correct.

A. Define a success in this context, and define the random variable X of interest. What is the probability of success?
B. What is the probability that Anna's first correct answer occurs on problem 5?
C. What is the probability that it takes more than 4 questions before Anna answers one correctly?
D. Construct a probability distribution table for X.
E. If Anna took a test like this many times and randomly guessed at each question, what would be the average number of questions she would have to answer before she answered one correctly?